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Acyclic Chromatic Index of Fully Subdivided Graphs and Halin Graphs

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An $acyclic$ edge coloring of a graph is a proper edge coloring such that there are no bichromatic cycles. The *acyclic chromatic index* of a graph is the minimum number k such that there is an acyclic edge coloring using k colors and is denoted by $a'(G)$. A graph G is called fully subdivided if it is obtained from another graph H by replacing every edge by a path of length at least two. Fully subdivided graphs are known to be acyclically edge colorable using $\Delta+1$ colors since they are properly contained in *2-degenerate graphs* which are acyclically edge colorable using $\Delta+1$ colors. Muthu, Narayanan and Subramanian gave a simple direct proof of this fact for the fully subdivided graphs. Fiamcik has shown that if we subdivide every edge in a cubic graph with at most two exceptions to get a graph G, then $a'(G) = 3$. In this paper we generalise the bound to Δ for all fully subdivided graphs improving the result of Muthu et al. In particular, we prove that if G is a fully subdivided graph and $\Delta(G) \geq 3$, then $a'(G) = \Delta(G)$.

Consider a graph $G = (V, E)$, with $E = E(T) \cup E(C)$ where T is a rooted tree on the vertex set V and C is a simple cycle on the leaves of T. Such a graph G is called a Halin graph if G has a planar embedding and T has no vertices of degree 2. Let K_n denote a complete graph on n vertices. Let G be a Halin graph with maximum degree Δ. We prove that,

$$a'(G) = \begin{cases} \Delta+2=5 & \text{if } G \text{ is } K_4 \\ \Delta+1=4 & \text{if } \Delta=3 \text{ and } G \text{ is not } K_4 \\ \Delta & \text{otherwise.} \end{cases}$$

Keywords: Acyclic edge coloring, acyclic edge chromatic number, subdivided graphs, Halin graphs.

1 Introduction

All graphs considered in this paper are finite and simple. A proper *edge coloring* of $G = (V, E)$ is a map $c : E \rightarrow C$ (where C is the set of available $colors$) with $c(e) \neq c(f)$ for any adjacent edges e,f. The minimum number of colors needed to properly color the edges of G is called the chromatic index of G and is denoted by $\chi'(G)$. A proper edge coloring c is called acyclic if there are no bichromatic cycles in the graph. In other words an edge coloring is acyclic if the union of any two color classes induces a set of paths (i.e., linear forest) in G. The *acyclic edge chromatic number* (also called *acyclic chromatic index*), denoted by $a'(G)$, is the minimum number of colors required to acyclically edge color G. The

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concept of *acyclic coloring* of a graph was introduced by Grünbaum [8]. The *acyclic chromatic index* and its vertex analogue can be used to bound other parameters like *oriented chromatic number* and *star chromatic number* of a graph, both of which have many practical applications, for example, in wavelength routing in optical networks ([3], [10]). Let $\Delta = \Delta(G)$ denote the maximum degree of a vertex in graph G. By Vizing's theorem, we have $\Delta \leq \chi'(G) \leq \Delta + 1$ (see [5] for proof). Since any acyclic edge coloring is also proper, we have $a'(G) \geq \chi'(G) \geq \Delta$.

It has been conjectured by Alon, Sudakov and Zaks [1] (and much earlier by Fiamcik [6]) that $a'(G) \leq \Delta + 2$ for any G. The best known result up to now for arbitrary graph, is by Ndreca et al. [13], who showed that $a'(G) \leq 9.62\Delta$.

Though the best known upper bound for general case is far from the conjectured $\Delta + 2$, the conjecture has been shown to be true for some special classes of graphs. Alon, Sudakov and Zaks [1] proved that there exists a constant k such that $a'(G) \leq \Delta + 2$ for any graph G whose girth is at least $k\Delta \log \Delta$. Muthu, Narayanan and Subramanian proved the conjecture for grid-like graphs [11]. In fact they gave a better bound of $\Delta + 1$ for these class of graphs. It has also been shown by Alon and Zaks [2] that determining whether $a'(G) \leq 3$ is NP-complete for an arbitrary graph G.

Our results: subdivided graphs

A graph G is called fully subdivided if it is obtained from another graph H by replacing every edge by a path of length at least two. Fully subdivided graphs are known to be acyclically edge colorable using $\Delta+1$ colors since they are properly contained in *2-degenerate graphs* which are acyclically edge colorable using $\Delta + 1$ colors [4]. Muthu, Narayanan and Subramanian [12] gave a simple direct proof of this fact for the fully subdivided graphs. Fiamcik [7] has shown that if we subdivide every edge in a cubic graph with at most two exceptions to get a graph G, then $a'(G) = 3$. In this paper we generalise the bound to Δ for all fully subdivided graphs improving the result of [12]. Note that the condition $\Delta \geq 3$ is necessary in all statements in Theorem 1 and Corollaries 1 and 2 below because a cycle is a subdivided graph with $\Delta = 2$ and it requires $3 = \Delta + 1$ colors.

First we prove the bound for a class of bipartite graphs and then obtain the result for fully subdivided graphs. Let $deg_G(x)$ denote the degree of the vertex x in graph G. We will abbreviate the notation to $deg(x)$ when the graph G is understood from the context.

Theorem 1 *Let $G = (V, E)$ be a bipartite graph with maximum degree $\Delta \geq 3$, and let $V = A \cup B$ be such that for any $a \in A$, $deg(a) = 2$. Then $a'(G) = \Delta$.*

Noting that fully subdivided graphs are a sub class of the bipartite graphs defined in Theorem 1, the following Corollary is obvious:

Corollary 1 *Let G be a fully subdivided graph of some graph H, where every edge is replaced by a path of length exactly two and $\Delta(G) \geq 3$. Then $a'(G) = \Delta(G)$.*

Remark Note that though graphs referred in the statement of Theorem 1 and Corollary 1 look the same, they are in fact different. Though a fully subdivided graph is a bipartite graph as defined in Theorem 1, it cannot have any cycle of length 4. Any cycle in a fully subdivided graph will be of length at least 6. But the bipartite graph we have considered in Theorem 1 can have cycles of length 4.

Corollary 2 *Let G be a fully subdivided graph of some graph H, where every edge is replaced by a path of length at least two and $\Delta(G) \geq 3$. Then $a'(G) = \Delta(G)$.*

Proof: Suppose every edge of H is replaced by a path of exactly two edges to get a graph G. Then by Corollary 1, we have $a'(G) = \Delta$. Thus there exists an edge uv in H which is replaced in G by a path of length at least three. Now let that path be $(u = a_0, a_1, a_2, \ldots, a_k, a_{k+1} = v)$. Now we contract the edge a_1a_2 to a_1 to get a graph G'. Graph G' is again a subdivided graph and by induction has an acyclic edge coloring using Δ colors. Let c' be such a coloring of G'. Now we obtain a coloring c of G from G' as follows: Let $c'(a_0a_1) = \alpha$, $c'(a_1a_3) = \beta$. Let γ be a color different from α and β. Now let $c(a_2a_3) = \beta$ and $c(a_1a_2) = \gamma$ and $c(e) = c'(e)$ for any other edge e of G. It is easy to see that the modified coloring is acyclic. It is proper since c' is proper and $c(a_0a_1), c(a_1a_2), c(a_2a_3)$ are all distinct. It is acyclic because if there is a bichromatic cycle created due to the recoloring, then it has to contain the edges a_0a_1, a_1a_2, a_2a_3. But we already know that all these three edges are colored differently. Thus c is acyclic. □

Our results: Halin Graphs

Consider a graph $G = (V, E)$, with $E = E(T) \cup E(C)$ where T is a rooted tree on the vertex set V and C is a simple cycle on the leaves of T. Such a graph G is called a Halin graph if G has a planar embedding and T has no vertices of degree $= 2$. The notion of Halin graphs was first used by Halin [9] in his study of minimally 3-connected graphs. Let K_n denote the complete graph on n vertices. In this paper, we prove the following theorem,

Theorem 2 *Let G be a Halin graph with maximum degree Δ. Then,*

$$a'(G) = \begin{cases} \Delta + 2 = 5 & \text{if } G \text{ is } K_4 \\ \Delta + 1 = 4 & \text{if } \Delta = 3 \text{ and } G \text{ is not } K_4 \\ \Delta & \text{otherwise.} \end{cases}$$

2 Preliminaries

Let $G = (V, E)$ be a simple, finite and connected graph of n vertices and m edges. Let $x \in V$. Then $N_G(x)$ will denote the neighbours of x in G. We abbreviate the notation to $N(x)$ when the graph G is understood from the context. Let $c : E \rightarrow \{0, 1, \ldots, k-1\}$ be an *acyclic edge coloring* of G. For an edge $e \in E$, $c(e)$ will denote the color given to e with respect to the coloring c. For $x, y \in V$, when $e = xy$ we may use $c(xy)$ instead of $c(e)$. For $S \subseteq V$, we denote the induced subgraph on S by $G[S]$.

We denote the set of colors in a coloring c by C. Let c be a partial coloring of G which uses at most $|C|$ colors. For any vertex $u \in V(G)$, we define $F_u(c) = \{c(e) | e = uv \in E(G)\}$. For an edge $ab \in E$, we define $S_{ab}(c) = F_b - \{c(ab)\}$. Note that $S_{ab}(c)$ and $S_{ba}(c)$ can be different. We will abbreviate the notation to F_u and S_{ab} when the coloring c is understood from the context.

Maximal bichromatic Path: Consider the subgraph induced by any two colors α and β with respect to any proper coloring c. The (α, β)-subgraph consists of even cycles, bichromatic paths of length at least two, isolated edges and isolated vertices. Now when we say maximal bichromatic path, we only concentrate on bichromatic paths of length at least two, ignoring the even bichromatic cycles, isolated edges and isolated vertices. Thus an (α,β) maximal bichromatic path with respect to a proper coloring c of G is a path-component of the (α, β)-subgraph that consists of at least two edges. An (α,β,a,b) maximal bichromatic path is an (α,β) maximal bichromatic path which starts at the vertex a with an edge colored α and ends at b. We emphasise that the edge of the (α,β,a,b) maximal bichromatic path incident on vertex a is colored α and the edge incident on vertex b can be colored either α or β. Thus the notations (α,β,a,b)

and (α,β,b,a) have different meanings. Also note that any maximal bichromatic path will have at least two edges. The following fact is obvious from the definition of acyclic edge coloring:

Fact 1 ([4]) *Given a pair of colors α and β of a proper coloring c of G, there can be at most one (α,β) maximal bichromatic path containing a particular vertex v, with respect to c.*

A color $\alpha \neq c(e)$ is a *candidate* for an edge *e* in G with respect to a partial coloring c of G if none of the adjacent edges of *e* are colored α. A candidate color α is *valid* for an edge *e* if assigning the color α to *e* does not create any bichromatic cycle in G.

Critical Path: Let $ab \in E$ and c be a partial coloring of G. Then a $(\alpha, \beta$,a,b) maximal bichromatic path which starts out from the vertex a via an edge colored α and ends at the vertex b via an edge colored α is called an (α, β, ab) critical path. Note that any critical path will be of odd length. Moreover the smallest length possible is three.

Fact 2 ([4]) *Let c be a partial coloring of G. A candidate color β is not $valid$ for the edge $e = (a, b)$ if and only if $\exists \alpha \in S_{ab} \cap S_{ba}$ such that there is a (α, β, ab) critical path in G with respect to the coloring c.*

Color Exchange: Let c be a partial coloring of G. Let $u, i, j \in V(G)$ and $ui, uj \in E(G)$. We define $Color\ Exchange$ with respect to the edge ui and uj, as the modification of the current partial coloring c by exchanging the colors of the edges ui and uj to get a partial coloring c', i.e., $c'(ui) = c(uj)$, $c'(uj) = c(ui)$ and $c'(e) = c(e)$ for all other edges e in G. The color exchange with respect to the edges ui and uj is said to be proper if the coloring obtained after the exchange is proper. The color exchange with respect to the edges ui and uj is $valid$ if and only if the coloring obtained after the exchange is acyclic.

Lemma 1 ([4]) *Let $u, i, j, a, b \in V(G)$, $ui, uj, ab \in E$. Also let $\{\lambda, \xi\} \in C$ such that $\{\lambda, \xi\} \cap \{c(ui), c(uj)\} \neq \emptyset$ and $\{i, j\} \cap \{a, b\} = \emptyset$. Suppose there exists an (λ,ξ,ab)-critical path that contains vertex u, with respect to a valid partial coloring c of G. Let c' be the partial coloring obtained from c by the color exchange with respect to the edges ui and uj. If c' is proper, then there will not be any (λ,ξ,ab)-critical path in G with respect to the partial coloring c'.*

3 Proof of Theorem 1

Let G be a minimum counterexample (with respect to the number of edges) for the theorem statement. It is easy to see that G is 2-connected since otherwise, the acyclic edge coloring of the blocks of G could be easily extended to G.

The graph $G = (V, E)$ is a bipartite graph with bipartition A, B and for any $a \in A$, $deg(a) = 2$. Let $a_1 \in A$. Let C be the set of colors. We have $|C| = \Delta(G)$. Now since G is a minimum counterexample, $G - \{a_1\}$ is acyclically edge colorable using at most Δ colors and let c'' be such a coloring of $G - \{a_1\}$. Let $N(a_1) = \{x, b_1\}$. Now we try to extend the coloring c'' to a valid coloring c of G. Now $C - F_{b_1}$ is the set of candidate colors for the edge b_1a_1. Assign a color from $C - F_{b_1}$ to the edge b_1a_1 to get a coloring c'. Without loss of generality, we assume that $c'(b_1a_1) = 1$.

Let $N(x) = \{a_1, a_2, \ldots, a_k\}$, where $k = deg(x)$ and $N(a_i) = \{x, b_i\}$. Suppose $deg(x) < \Delta$. Then if $c'(b_1a_1) \notin F_x$, there exists a candidate color for the edge xa_1. It is also valid since if there is a bichromatic cycle created due to this coloring then it has to pass through the vertex x, implying that $c'(b_1a_1) \in F_x$, a contraction to our assumption. On the other hand if $c'(b_1a_1) \in F_x$, then there are at least two candidate colors for the edge xa_1, say α and β. If both of them are not valid then by Fact 2 there exist $(1, \alpha, xa_1)$ and $(1, \beta, xa_1)$ critical paths with respect to c'. Since $c'(b_1a_1) \in F_x$, without loss of generality let $c'(xa_2) = 1$. Then the above mentioned critical paths have to pass through a_2, implying that $1, \alpha, \beta \subseteq F_{a_2}$, a contradiction since $deg(a_2) = 2$. Thus at least one of α or β is valid for the edge xa_1.

Thus $deg(x) = \Delta$. Now we have the following cases:

case 1: $c'(b_1a_1) \in F_x$

Without loss of generality let $c'(xa_2) = c'(b_1a_1) = 1$. Now since $deg(x) = \Delta$, there exists only one candidate color for the edge a_1x. Let it be α. Now if the color α is not valid, then by Fact 2, $c(a_2b_2) = \alpha$ and there exists a $(1, \alpha, a_1x)$ critical path containing the edge a_2b_2. If there exists a vertex a_i other than a_1, a_2 such that $c'(a_ib_i) = \alpha$, then exchange the colors of edges xa_2 and xa_i. Now by Lemma 1, the $(1, \alpha, a_1x)$ critical path is broken and color α is valid for the edge a_1x. If there is no such vertex a_i, we have $c(a_ib_i) \neq \alpha$ for all $3 \leq i \leq k$. Let $c'(xa_3) = \beta$. Now recolor the edge xa_3 using color α to get a coloring d. The coloring d is proper since $\alpha \notin F_x(c')$ and $c'(a_3b_3) \neq \alpha$. It is also acyclic because if there is a bichromatic cycle created, then it has to be a $(1, \alpha)$-bichromatic cycle, implying that there existed a $(1, \alpha, xa_3)$ critical path with respect to c', a contradiction to Fact 1 since there already existed a $(1, \alpha, a_1x)$ critical path in c'. Thus d is acyclic. Now we get a coloring c from d by assigning $c'(xa_3) = \beta$ for the edge xa_1. The color is valid since any bichromatic cycle if present has to involve edges a_1x, xa_2, a_2b_2 which are colored differently. Thus we have a valid coloring of G, a contradiction for the fact that G is a counterexample.

case 2: $c'(b_1a_1) \notin F_x$

This means that $\{c'(b_1a_1) = 1\} \cup F_x = C$. We have the following two subcases:

2.1: $c'(a_ib_i) \neq 1$ for some i, $2 \leq i \leq k$.

Suppose without loss of generality that $c'(a_ib_i) = \alpha \neq 1$ and $c'(xa_i) = \beta$. Then recolor edge xa_i using color 1 to get a coloring d. If the coloring d is valid we are in *case 1*. Otherwise a bichromatic cycle has been created due to this recoloring, say a $(1, \alpha)$-bichromatic cycle going through b_i, a_i, a_j, b_j (for some $j \neq i$) with $d(xa_j) = \alpha$ and $d(a_jb_j) = 1$. Recolor xa_j using color β to get a valid coloring d' and we are in *case 1*.

2.2: $c'(a_ib_i) = 1$ for all i, $2 \leq i \leq k$.

Without loss of generality we can assume that $c'(xa_2) = 2$. Now recolor edge xa_1 using color 2, edge xa_2 using color 1 and discard the color of edge a_2b_2 to get a coloring d. Now again this situation is similar to assigning color to edge xa_1, here b_2 and a_2 taking the role of x and a_1 respectively. The vicinity of b_2 will be similar to x. Thus we are again in the same cases as in the case of x. Either we can extend the coloring as in the cases described above or we are in the case where $1 \notin F_{b_2}(d)$, for $y_i \in N_G(b_2)$, $1 \in F_{y_i}(d)$ and $\{d(xa_2) = 1\} \cup F_{b_2}(d) = C$. Hence $2 \in F_{b_2}(d)$. We move on like this along the $(1, 2)$-bichromatic path that was in the coloring c' by iterating the previous recoloring procedure.

Since c' was acyclic we cannot come back to a vertex which we had already visited before. We now want to prove the existence of a vertex $z \in B$ such that for some vertex $w_i \in N(z)$,

$1 \notin F_{w_i}(d)$ and so we can extend the coloring to the whole graph G using *subcase 21*. Suppose by contradiction we visited all vertices of B and come back to x via b_1, we know that with respect to c', $c'(a_1b_1) = 1$ and $c'(xa_2) = 2$. This means that there exists a $(1, 2, a_1, x)$ bichromatic path which is of even length. This path along with the edge xa_1, will give an odd cycle, a contradiction since G is bipartite. We can infer that there exists a vertex z such that $1 \notin F_{w_i}(d)$, for some $w_i \in N(z)$. Now we are in *subcase 21*. Thus we have a coloring for G using Δ colors and hence a contradiction to our assumption that G is the minimum counterexample.

4 Proof of Theorem 2

Let $G = (V, E)$ be a simple, finite and connected Halin graph of n vertices and m edges. For a rooted tree T, the $depth$ of the root is taken to be 0 and the $depth$ of any other vertex is one greater than the $depth$ of its parent. The depth of the tree is taken to be the maximum depth of a vertex in it. If a Halin graph has an acyclic edge coloring using exactly the number of colors specified in Theorem 2, then the graph is said to have a *desired coloring*.

Note that there is a planar embedding for the Halin graph. For the proof we choose a maximum degree vertex of T to be the root. This ensures that the root is not a leaf. The cycle C is passing through the leaves of the tree T. Let σ be a circular ordering of the leaves of T, obtained as we traverse the cycle C in the clockwise direction.

Claim 1 *Halin graph of depth* 1 *has a desired coloring.*

Proof: Let G be a Halin graph on n vertices. Let u be the root. If $G = K_4$, we know that H is acyclically edge colorable using 5 colors. Since $G \neq K_4$, clearly $\Delta(G) \geq 4$. Let $a_0, a_1, \ldots, a_{\Delta-1}$ be the children of u ordered with respect to the circular ordering σ. We color the graph G as follows:

- $c(u, a_i) = i$ for $0 \leq i \leq \Delta - 1$.
- $\forall i,\ 0 \leq i \leq \Delta - 1$, $c(a_i a_{i+1 \pmod{\Delta}}) = c(u a_{i+2 \pmod{\Delta}})$.

It is easy to check that the coloring is proper and acyclic. □

Let u be an internal vertex of T with maximum depth. In view of Claim 1, we can assume that the depth of T is more than 1 and thus u is not the root. Let v be the parent of u. Since u is an internal vertex of T with maximum depth it is easy to see that the children of u have to be consecutive with respect to the circular ordering σ (Recall the way the ordering was defined). Let $a_0, a_1, \ldots, a_{k-1}$, $2 \leq k \leq \Delta - 1$ be the children of u ordered with respect to the circular ordering σ. Let x be the predecessor of a_0 and y be the successor of a_{k-1} with respect to the ordering σ. That is $N_G(a_0) = \{x, a_1, u\}$ and $N_G(a_{k-1}) = \{y, a_{k-2}, u\}$. It is easy to see that since u is not the root there are at least 2 more leaves in T, in addition to $a_0, a_1, \ldots, a_{k-1}$. Therefore $x \neq y$.

We prove Theorem 2 by induction on the number of vertices. Let G be a Halin graph on n vertices. Now we get a Halin graph G' by contracting the subset $\{u, a_0, a_1, \ldots, a_{k-1}\}$ of V. Note that since the vertex with maximum degree of G was chosen as the root and depth of T is more than 1, we have $\Delta(G') = \Delta(G) = \Delta$. Let z be the vertex resulting from this contraction. Then it is easy to see that $N_{G'}(z) = \{v, x, y\}$.

Now by induction hypothesis G' has a $desired\ coloring$. We consider two cases:

case 1: $G' = K_4$.

Here we have $\Delta = 3$. In this case it is easy to verify that G has to be isomorphic to the graph given in Figure 1. Thus it is sufficient to show that this graph can be acyclically edge colored using 4 colors. The coloring is explicitly given in Figure 1.

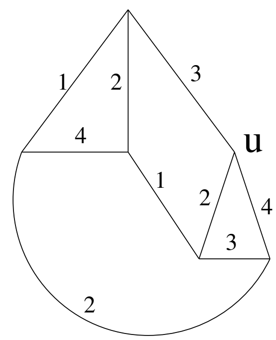

Fig. 1: Coloring of G when $G' = K_4$

case 2: $G' \neq K_4$.

Note that in G' the edges zv, zx and zy are mutually adjacent and hence the colors on them have to be different. We can also assume that the colors on the edges zv, zx, zy are $\Delta - 1, 0, 1$ respectively since otherwise we can rename the colors as required.

We get the coloring d of G from c of G' as follows:

- $d(xa_0) = c(xz) = 0$, $d(a_{k-1}y) = c(zy) = 1$.
- $d(ua_0) = 1$, $d(ua_1) = 0$ and $d(ua_i) = i$ for $2 \leq i \leq k-1 \leq \Delta - 2$.
- $\forall i,\ 0 \leq i \leq k-3$, $d(a_i a_{i+1}) = d(ua_{i+2})$.
- $d(a_{k-2}a_{k-1}) = c(zv) = \Delta - 1$.
- $d(uv) = c(zv) = \Delta - 1$.
- $d(e) = c(e)$ for all other edges e.

Claim 2 *The coloring* d *is acyclic.*

Proof: It is clear that the coloring is proper. We claim that this coloring is acyclic also. First observe that there cannot be a bichromatic cycle that contains any one of the edges uv, xa_0 or ya_{k-1}. This is because if there is a bichromatic cycle containing any one of these edges then it has to contain at least two edges from the three mentioned. This implies that the bichromatic cycle existed even in G' since these edges were present in G' too as zv, zx and zy with the same colors (Recall that x, y, v are distinct vertices in G').

Thus we can infer that if there is a bichromatic cycle then it has to be formed in the subgraph induced by the vertices $u, a_0, a_1, \ldots a_{k-1}$. Now with respect to this induced subgraph each color class has at most two edges and thus if there is a bichromatic cycle, it has to be a 4-cycle. In this induced subgraph the color classes 0, 1 and $\Delta - 1$ have exactly one edge each. Thus there cannot be a bichromatic cycle containing any of these colors. Now suppose there is a (i, j) bichromatic cycle with $i > j$ and $i, j \notin \{0, 1, \Delta - 1\}$. Note that $2 \leq j$. Then the edges have to be ua_i, ua_j and the cycle edges between them. It means that the cycle edges are adjacent, which means that $i - j = 2$. Hence we have $d(a_j a_{j+1}) = i$ and $d(a_{j+1}a_i) = j$. Since $i \leq k-1$, we infer that $j + 1 \leq k - 3$. Therefore by the definition of the coloring d, $d(a_{j+1}a_i) = j + 3 > j$, a contradiction. Thus the (i, j)-bichromatic cycle does not exist and thus coloring c is acyclic. □

Therefore G has a desired coloring.

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